Universal Related-Key Linear Hull Distinguishers for Key-Alternating Block Ciphers

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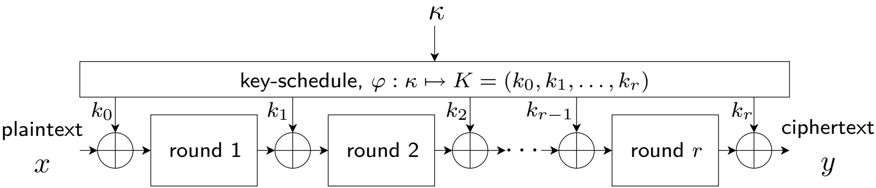
Key-Alternating Block Ciphers

AES, Serpent, PRESENT, ...



- κ = user-supplied key
- K = expanded key
- k_i = round subkeys
- n = block size in bit

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Key Observation

$$(-1)^{U_i \diamond K} - (-1)^{U_i \diamond K'} = \begin{cases} 0, & \text{if } U_i \diamond K = U_i \diamond K' \Leftrightarrow U_i \diamond \Delta = 0 \\ \pm 2, & \text{if } U_i \diamond K \neq U_i \diamond K' \Leftrightarrow U_i \diamond \Delta = 1 \end{cases}$$

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  - M growing exponentially in HW(active positions of Δ)

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- Complexity:
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  - to a large extent independent of $\#$ rounds